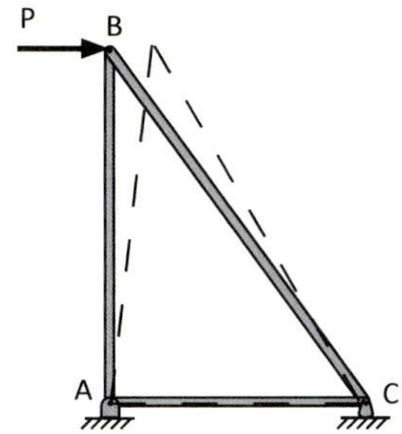
Chapter 9 - Simple Stresses
Reading: Chapter 9 - Pages 323—337

9-1
Introduction

The intensities of the internal resisting force are called stresses.



Statics
All bodies are assumed to be rigid.

Strength of Materials
Bodies are considered deformable.
Deformation per unit length is called the strain.

Two properties that affect how a body (member) will react to loading:
1. The Material
2. Shape

Definitions:

Strength	Load carrying capacity based on stresses inside a member
Stiffness	Ability to resist deformation. Deformation characteristics. (Serviceability)
Stability	The ability of a slender member to maintain its initial configuration without buckling while being subjected to compressive loading.

9-2
Normal and Shear Stresses

Stress ≡ the intensities of internal forces per unit area

There are two types of stresses:
1. Normal Stresses (σ) - are caused by internal forces normal (perpendicular $\perp$) to the area in question.
2. Shear Stresses (τ) - are caused by internal forces tangential (parallel $=$) to the area under question.

- Stresses are the most important concepts in the study of strength of materials.
- Whenever a body is subjected to external loads, stresses are induced within the body.
- Whether the material will fail and to what extent it will deform depends on the amount of stresses induced within the body.

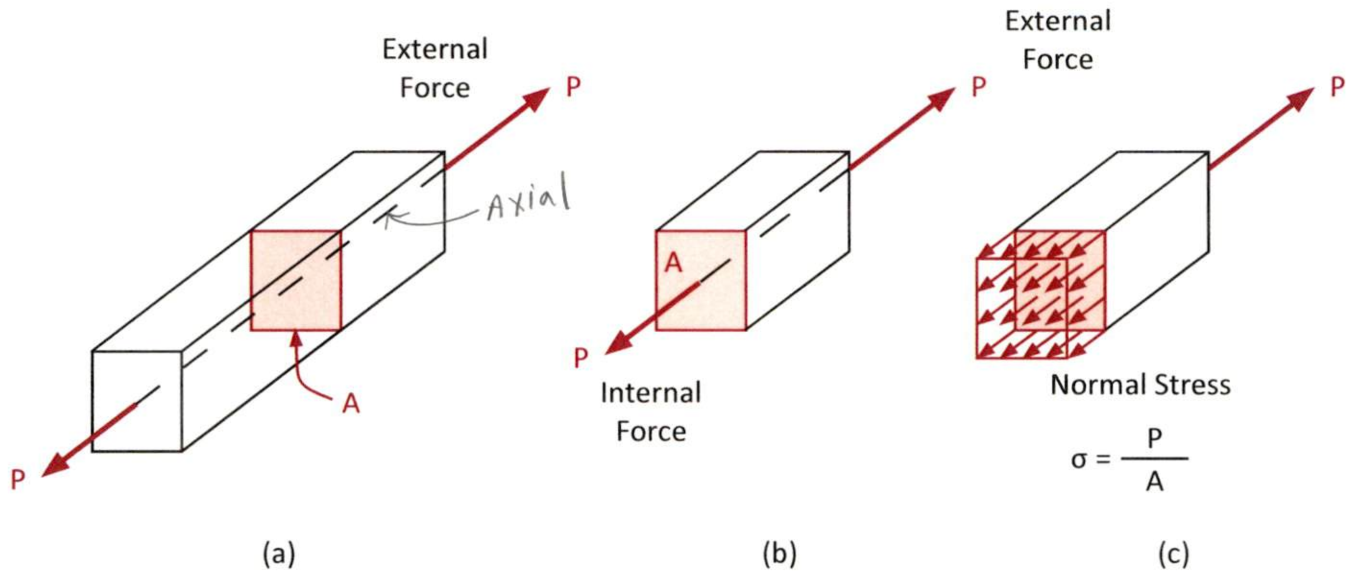
Units of Stress

U.S. Customary Units
Pounds per Square Inch (psi)
Pounds per Square Foot (psf)
Kips per Square Inch (Ksi)

S.I. Units
Newton's per square meter (N/m^2)
N/m^2 = Pascal (Pa)

1 kPa = 10^3 Pa
1 MPa = 10^6 Pa
1 GPa = 10^9 Pa

Normal Stress (σ) - intensity of internal force perpendicular ⊥ to the area under question.



(a) If the bar is in equilibrium, any segment must be in equilibrium when cut by a transverse plane.

(b) Equilibrium conditions requires that the internal force in the section be equal to the external force P.

(c) The internal force is normal (perpendicular ⊥) to the section, the stress induced is the normal stress.

Normal stresses due to axial loads through the centroids of the cross-sections are usually distributed uniformly over a cross section.

Direct Normal Stress Formula

$$\sigma = \frac{P}{A}$$

Where σ = the normal stress in the cross-section
P = the internal axial force at the section
A = the cross-sectional area of the rod

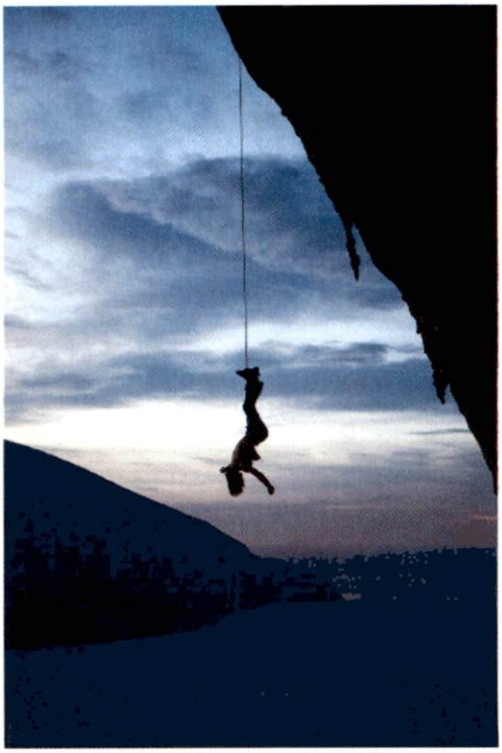
Tensile Stress - induced by tensile forces (T)
Compressive Stress - induced by compressive forces (C)

Tension (T) — Member is elongating (stretching)

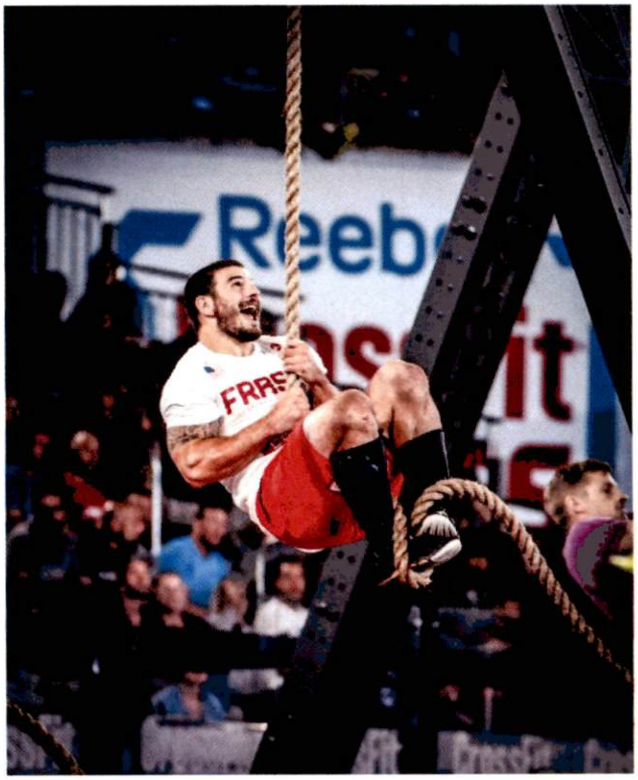
Compression (C) — Member is shortening

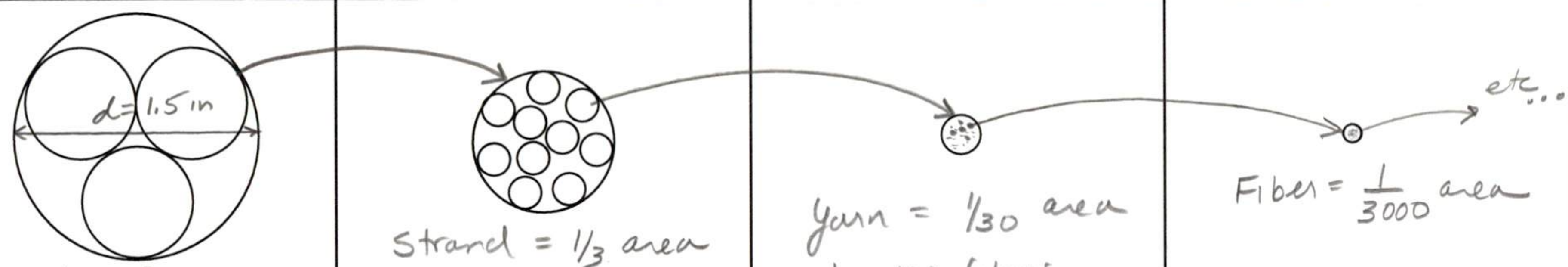
Rope Construction

- Rope is sometimes generally referred to as cordage.
- Rope construction involves twisting fibers together to form yarn. For twisting rope, the yarn is then twisted into strands, and the strands twisted into rope.
- Three-stranded twisted rope is the most common construction.
- For braided rope, the yarn is braided rather than being twisted into strands.
- Double-braided rope has a braided core with a braided cover.
- Plaited rope is made by braiding twisted strands.
- Other rope construction includes combinations of these three techniques such as a three-strand twisted core with a braided cover.
- The concept of forming fibers or filaments into yarn and yarn into strands or braids is fundamental to the rope-making process.



A 1-½ in diameter fitness rope is composed of three strands. Each strand is made of 10 yarns and each yarn is made of 100 fibers. The tension in the rope is 240 lb. Determine the normal stress in the rope, strand, yarn, and fiber.





Rope	Strand	Yarn	Fiber
Area $= \pi \frac{(1.5\,in)^2}{4} = 1.767\,in^2$	Strand = 1/3 area with 10 yarn	yarn = 1/30 area with 100 fibers	Fiber $= \frac{1}{3000}$ area
Rope Holds	Strand Holds	yarn Holds	Fiber Holds
Force = 240 lb	Force $= \frac{240\,lb}{3} = 80\,lb$	Force $= \frac{80\,lb}{10} = 8\,lb$	Force $= \frac{8\,lb}{100} = 0.08\,lb$
$\sigma = \frac{240\,lb}{1.767\,in^2} = 136\,psi$	$\sigma = \frac{80\,lb}{\frac{1}{3}(1.767\,in^2)} = 136\,psi$	$\sigma = \frac{8\,lb}{\frac{1}{30}(1.767\,in^2)} = 136\,psi$	$\sigma = \frac{0.08\,lb}{\frac{1}{3000}(1.767\,in^2)} = 136\,psi$

The Normal stress Stays constant!

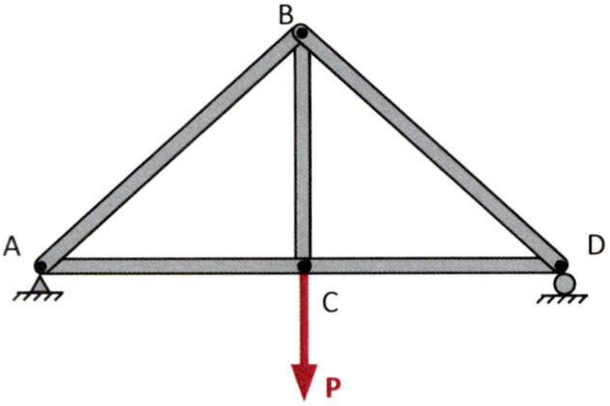
Normal stress

$$\sigma = \frac{P}{A}$$

A → ⊥ to axial Force

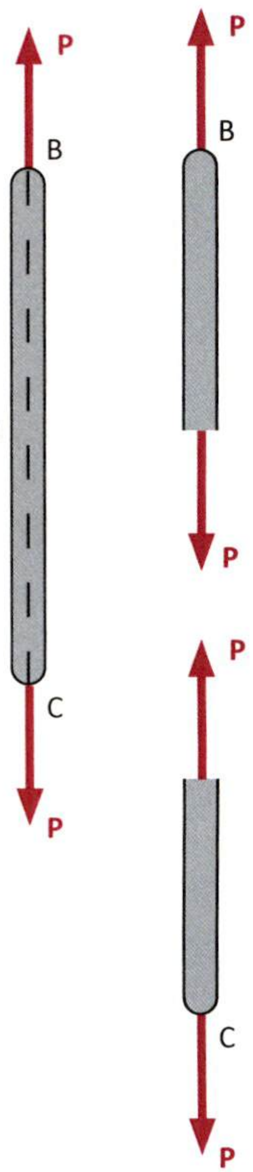
Allowable Axial Load

Consider the simply supported truss shown.



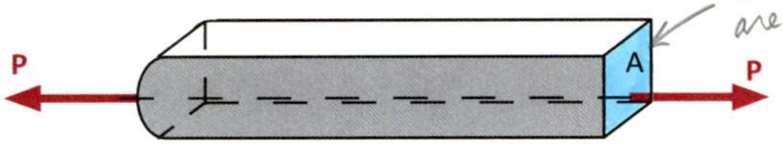
Member BC is in axial tension

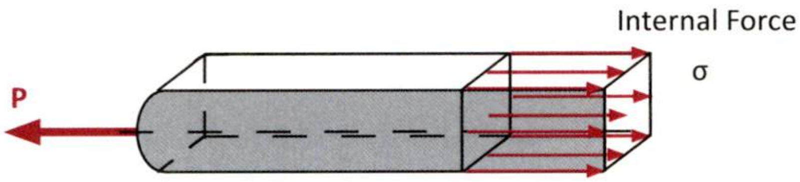
Cut the member along its length



Rotate the member 90°

cross-sectional area ⊥ to force P





Internal Normal Stress ≡ $\sigma = \dfrac{P}{A}$ [Intensity of Force / unit area]

Also, $P = \sigma A$

≡ Resultant of stress [product of the stress x Area = Force]

Upper Limit of Allowable Stress

If we know the strength of a material, $\sigma_{Failure}$, can divide by a Factor of Safety (F.S. > 1) to give safe or allowable stress,

$$\sigma_{Allow} = \sigma_{Failure} \,/\, \text{F.S.}$$

the allowable force, $P_{Allow} = \sigma_{Allow} A$ (analysis)

Required Area

Required minimum cross-sectional area A of a member designed to carry a maximum axial load P without exceeding the allowable stress σ_{allow}.

$$A = \frac{P}{\sigma_{Allow}}$$ (Design)

These two equations apply only if the compressive member length is relatively short compared to the lateral dimensions of the member. For now, the discussion is limited to short compression members that do not buckle.

Internal Axial Force Diagram

Variation of internal axial force along the length of a member can be depicted by an internal axial force diagram whose ordinate at any section of a member is equal to the value of the internal axial force at that section.

<u>Convention Used in Textbook</u>

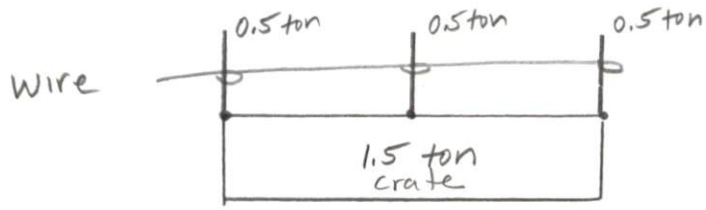
Tensile force is plotted as postive

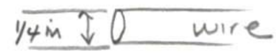
Compressive force is plotted as negative

Example 1

9–1 A 1.5-ton crate is hoisted by three steel wires. Each wire is $\frac{1}{4}$ in. in diameter and each carries one-third of the load. Determine the stress in the wires.

Solution.





1 ton = 2000 lb

$$\sigma_{allow} = \frac{P}{A}$$

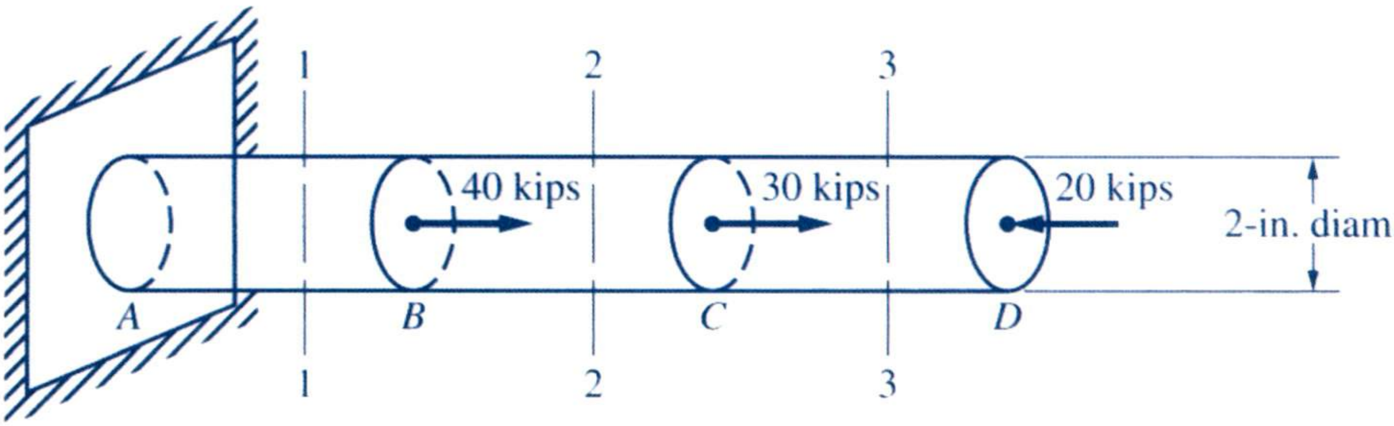
$$= \frac{0.5 \text{ ton} \times 2000 \frac{\text{lb}}{\text{ton}}}{\frac{\pi (1/4 \text{ in})^2}{4}}$$

$$= \frac{1000 \text{ lb}}{0.0491 \text{ in}^2}$$

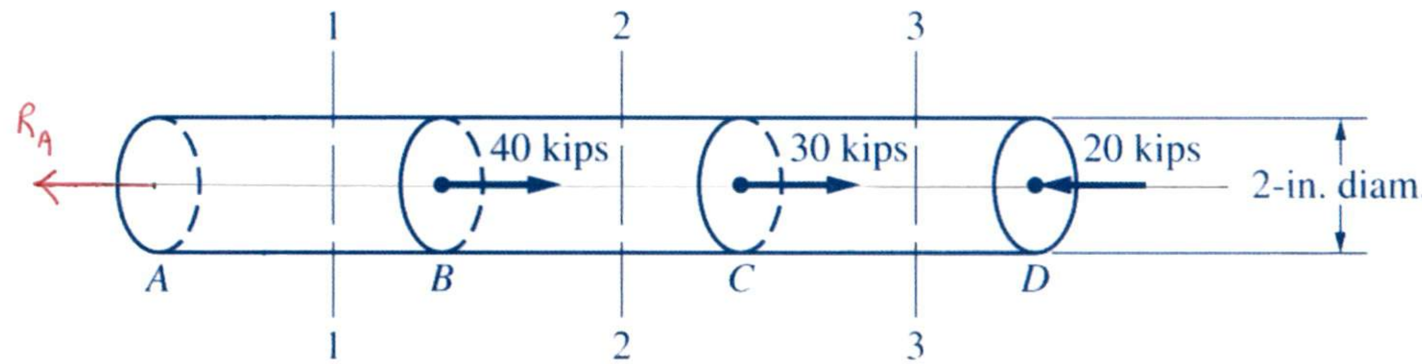
$$= \underline{\underline{20{,}370 \text{ psi}}} \quad (T) \quad (\text{each wire})$$

Example 2

9–3 to **9–5** Refer to Figs. P9–3 to P9–5. Plot the internal axial force diagram and determine the normal stresses in segments AB, BC, and CD of each member due to the axial loads shown.



Solution.



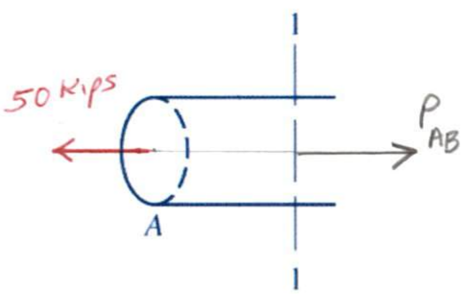
FBD - Entire Rod

Equilibrium Equations

$$[\Sigma F_x = 0] \qquad -R_A + 40\text{ kips} + 30\text{ kips} - 20\text{ kips} = 0$$

$$R_A = \underline{\underline{50\text{ kips}}} \leftarrow$$

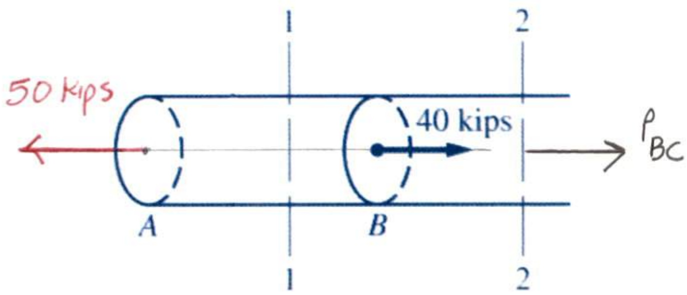
If the rod is in equilibrium, each section of the rod is in equilibrium.



$$[\Sigma F_x = 0] \qquad -50\text{ kips} + P_{AB} = 0$$

$$P_{AB} = \underline{\underline{50\text{ kips}}}\ (T)$$

FBD - Left Portion of Section 1-1

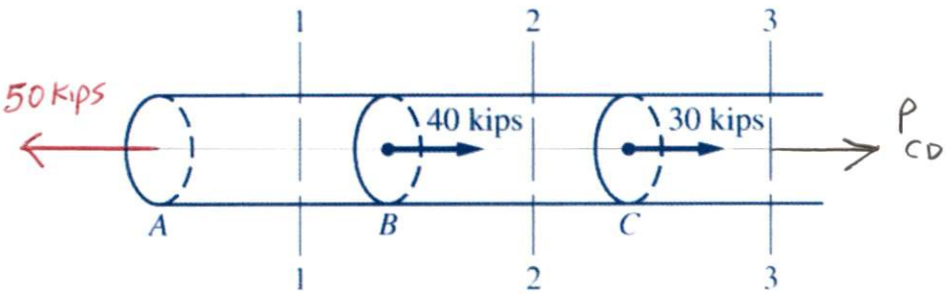


FBD - Left Portion of Section 2-2

$[\Sigma F_x = 0]$

$-50\text{ kips} + 40\text{ kips} + P_{BC} = 0$

$P_{BC} = \underline{10\text{ kips}}\ (T)$



FBD - Left Portion of Section 3-3

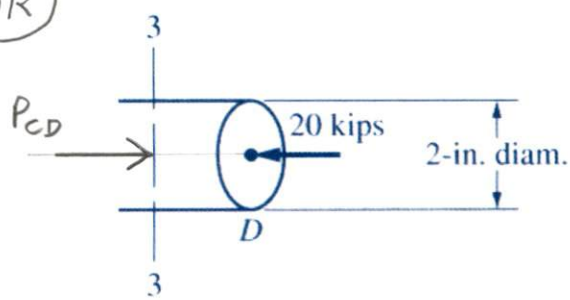
$[\Sigma F_x = 0]$

$-50\text{ kips} + 40\text{kips} + 30\text{kips} + P_{CD} =$

$P_{CD} = -20\text{ kips}\ (T)$

and $\boxed{P_{CD} = 20\text{ kips}\ (C)}$

OR



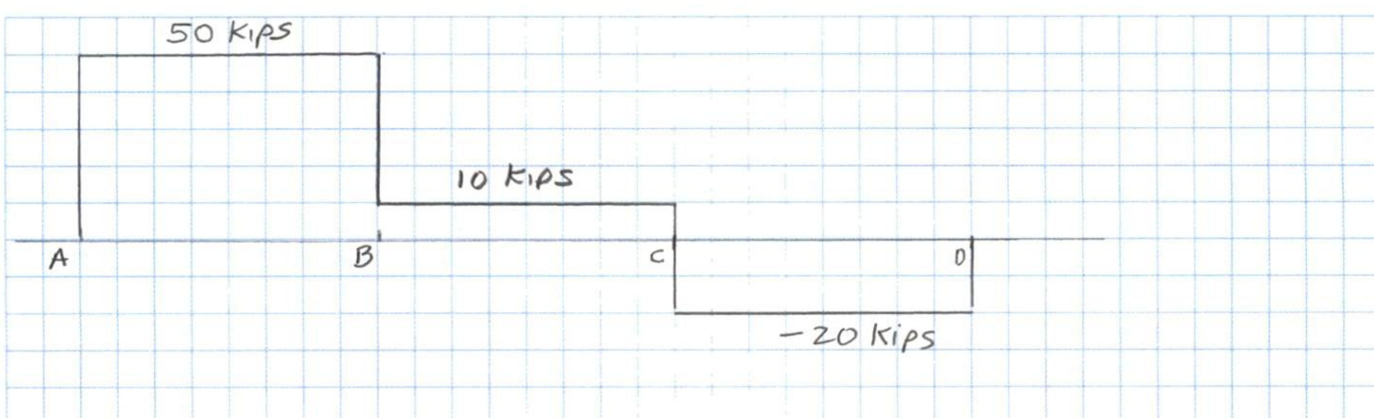
FBD - Right Portion of Section 3-3

$[\Sigma F_x = 0]$

$P_{CD} - 20\text{ kips} = 0$

$P_{CD} = \underline{20\text{ kips}}\ (C)$

Internal Axial Force Diagram



Normal Stresses in Each Segment

$$\sigma_{AB} = \frac{50 \text{ kips}}{\frac{\pi (2 \text{in})^2}{4}} = +15.92 \text{ ksi } (T)$$

$$\sigma_{BC} = \frac{10 \text{ kips}}{3.14 \text{ in}^2} = +3.18 \text{ ksi } (T)$$

$$\sigma_{CD} = \frac{-20 \text{ kips}}{3.14 \text{ in}^2} = -6.37 \text{ ksi } (C)$$

9–6 A short column composed of two standard steel pipes is subjected to a load $P = 20$ kips, as shown in Fig. P9–6. Determine the compressive stress in each pipe. Neglect the weight of the pipes.

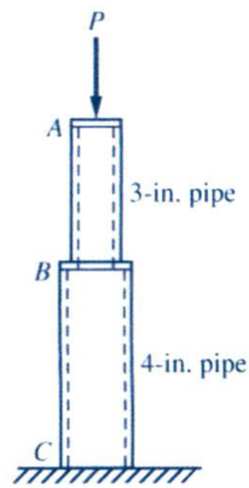


FIGURE P9–6

TABLE A–5(a) Properties of Structural Steel Pipes: U.S. Customary Units

Nominal Diameter (in.)	Outside Diameter d_o (in.)	Inside Diameter d_i (in.)	Wall Thickness t (in.)	Weight per ft w (lb/ft)	Properties A (in.^{2})	I (in.^{4})	S (in.^{3})	r (in.)
Standard Weight								
$\frac{1}{2}$	0.840	0.622	0.109	0.85	0.250	0.017	0.041	0.261
$\frac{3}{4}$	1.050	0.824	0.113	1.13	0.333	0.037	0.071	0.334
1	1.315	1.049	0.133	1.68	0.494	0.087	0.133	0.421
$1\frac{1}{4}$	1.660	1.380	0.140	2.27	0.669	0.195	0.235	0.540
$1\frac{1}{2}$	1.900	1.610	0.145	2.72	0.799	0.310	0.326	0.623
2	2.375	2.067	0.154	3.65	1.07	0.666	0.561	0.787
$2\frac{1}{2}$	2.875	2.469	0.203	5.79	1.70	1.53	1.06	0.947
3	3.500	3.068	0.216	7.58	2.23	3.02	1.72	1.16
$3\frac{1}{2}$	4.000	3.548	0.226	9.11	2.68	4.79	2.39	1.34
4	4.500	4.026	0.237	10.79	3.17	7.23	3.21	1.51

$$\sigma_{AB} = \frac{P}{A} = \frac{-20 \text{ kips}}{2.23 \text{ in}^2} = -8.97 \text{ ksi } (C)$$

$$\sigma_{BC} = \frac{P}{A} = \frac{-20 \text{ kips}}{3.17 \text{ in}^2} = -6.31 \text{ ksi } (C)$$